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A statistical identity linking folded and censored distributions

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Abstract

This paper models expected future values of Gaussian stochastic processes that are bounded by reflecting barriers. Such expectations are of course crucial to any model with forward looking agents. The approach is illustrated by applying it to an exchange rate target zone. By adopting a distributional approach, the formal analysis can be both simple and somewhat elegant. In doing so, we show that the first moments of folded and censored distributions are related in a surprisingly neat way. The setting is discrete-time, though where appropriate we extend the analysis to the continuous-time analogue of reflected Brownian motion.

Key words: Censored/folded distributions; Random walks; Reflecting barriers; Exchange rate target zone
JEL classification: C10; F31

1. Introduction

Many recent innovations in economic theory share, as a common underlying element, a stochastic process that is bounded by a reflecting barrier. Examples include such major growth areas as the literature on exchange rate target zones and the theory of irreversible investment under uncertainty – Dixit (1993) and Rose (1993) consider a variety of such problems. In the statistical literature, the term 'reflecting barrier' has two main interpretations. The first interpretation can be captured by a censored distribution, while the second can be captured by

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a folded distribution. Given this backdrop, Section 2.1 generates a new statistical identity by illustrating that the first moments of censored and folded distributions are related in a surprisingly neat way, and hence that the two types of reflecting barrier are also related. Section 2.2 generates explicit solutions when the unbounded stochastic process follows a Gaussian random walk. Section 3 applies the above methodology to the problem of modelling an exchange rate in a perfectly credible target zone. By adopting a statistical approach, we illustrate that this problem is in fact rather simple. At the same time, the analysis is general for it can apply to both discrete-time and continuous-time settings. The paper adopts a discrete-time setting, but considers the continuous-time analogue where appropriate.

2. Formal analysis

2.1. *Reflecting barriers and censored/folded distributions*

Let $\{\varepsilon_t\}$ denote a shock-generating process of independent, identically distributed random variables. In the *absence* of any barriers, this yields a random process defined recursively by

$$x_{t+1}^* = x_t^* + \varepsilon_{t+1}.$$

More generally, given some drift term k, we have[1]

$$x_{t+1}^* = x_t^* + k + \varepsilon_{t+1}.$$

In the *presence* of lower and/or upper reflecting barriers denoted $\underline{x}$, $\bar{x}$, respectively, a reflected process $\{x_t\}$ will be obtained in contrast to the unrestricted process $\{x_t^*\}$ described above. At any time t, outcome x_t is assumed known, whereas x_{t+1} is of course unknown. In any model with forward-looking agents, the expectation of x_{t+1} conditional on x_t will then, typically, be crucial. For notational convenience, let $X = x_t + k + \varepsilon_{t+1}$. Thus, X denotes next period's outcome, in the absence of a barrier. Given x_t, let X have pdf $\phi(X)$ with mean μ, and distribution function $\Phi(X)$.

In the statistical literature, the term *reflecting barrier* takes on two distinct interpretations. To distinguish between them, we refer to them as the Reflecting *Sticky* Barrier and the Reflecting *Mirror* Barrier.

[1] Concomitant to the discrete-time framework, the drift term k and the shock term ε are assumed to be contemporaneous.

The reflecting sticky barrier

Cox and Miller (1965) define a reflecting barrier as 'a state which, when crossed in a given direction, say downwards, holds the particle until a positive jump occurs and allows the particle to move up and resume the random walk' (p. 24; also see p. 61). Similarly, a particle crossing an *upper* barrier will be held at this barrier until a negative shock occurs. We shall refer to such states as reflecting *sticky* barriers. If $\underline{x}$, $\bar{x}$ denote *sticky* barriers, then the reflected process $\{x_t\}$ is defined recursively by

$$x_{t+1} = \begin{cases} \bar{x} & \text{if } X \geq \bar{x}, \\ X & \text{if } \underline{x} < X < \bar{x}, \\ \underline{x} & \text{if } X \leq \underline{x}. \end{cases} \tag{1}$$

Thus, x_{t+1} has a doubly censored distribution. Let η_{DC} (*doubly censored*) denote the expectation of x_{t+1}. Then trivially by (1):

$$\eta_{DC} = \int_{-\infty}^{\underline{x}} \underline{x}\phi(X)\,\mathrm{d}X + \int_{\underline{x}}^{\bar{x}} X\phi(X)\,\mathrm{d}X + \int_{\bar{x}}^{\infty} \bar{x}\phi(X)\,\mathrm{d}X, \tag{2}$$

which we shall refer to later.

The reflecting mirror barrier

A second (and more common) definition of a 'reflecting barrier' treats the barrier as if it were a mirror reflecting a light wave – see for instance Karlin and Taylor (1981, p. 251). Fig. 1 illustrates the concept with a reflecting upper barrier.

At time t, x_t is given (point P). In the *absence* of a barrier, $x_{t+1} = X$ may be located anywhere along line LL with density $\phi(X)$. In the *presence* of a reflecting mirror barrier at $\bar{x}$, an outcome $X = Q_1$ (inside the band) will be unaffected by that barrier. However, a '*virtual*' outcome $X = Q_2$ will be reflected and will end up at $Q_3 = Q_2 - 2(Q_2 - \bar{x}) = 2\bar{x} - Q_2 = 2\bar{x} - X$ (the '*real*' outcome). In this vein, if $\underline{x}$ and $\bar{x}$ denote reflecting *mirror* barriers, a reflected process $\{x_t\}$ will now be defined recursively by

$$x_{t+1} = \begin{cases} 2\bar{x} - X & \text{if } X \geq \bar{x}, \\ X & \text{if } \underline{x} < X < \bar{x}, \quad \text{assuming no secondary reflection.} \\ 2\underline{x} - X & \text{if } X \leq \underline{x}, \end{cases} \tag{3}$$

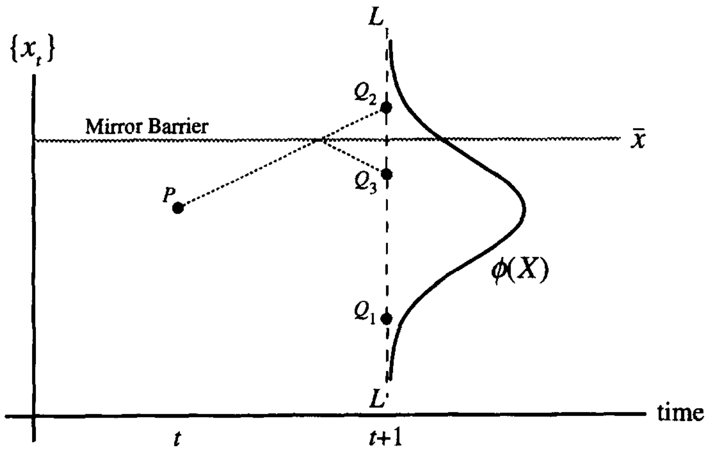

Fig. 1. Reflecting upper barrier.

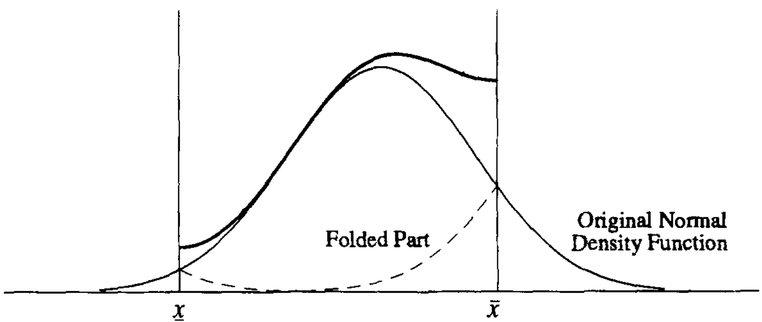

Fig. 2. Generalised doubly folded normal density function.

In the presence of both an upper *and* a lower reflecting *mirror* barrier, secondary reflection will occur if X is so large that not only is it reflected off the upper barrier, but that it then bounces across the entire band, and reflects off the lower barrier as well. If for instance $\phi(X)$ is normal, this assumption of 'no secondary reflection' will be reasonable if (i) $\mu \in [\underline{x}, \bar{x}]$ and (ii) $\bar{x} - \underline{x} \geq 3\sigma$ (i.e., the band is at least three standard deviations wide). Under these conditions, the probability of secondary reflection is approximately zero.[2] Secondary reflection is discussed further in Appendix B where condition (i) is relaxed.

Graphically, Eq. (3) states that the transition density $\phi(X)$ between the barriers is unaffected by those barriers, whilst the density lying outside the barriers will get reflected into the interior of the barrier band. Fig. 2 illustrates

[2] At worst, if $\mu = \underline{x}$ or $\bar{x}$ (see Fig. 2), the Prob[secondary reflection] $= \Phi(\mu - 3\sigma) = 0.0013$.

when the transition density $\phi(X)$ is normal. We term the resulting distribution to be the *Generalised Doubly Folded Normal Distribution*.[3]

Let λ_{DF} (*doubly folded*) denote the expectation of x_{t+1}. Then, by (3),

$$\lambda_{DF} = \int_{-\infty}^{\underline{x}} (2\underline{x} - X)\phi(X)\,\mathrm{d}X + \int_{\underline{x}}^{\bar{x}} X\phi(X)\,\mathrm{d}X + \int_{\bar{x}}^{\infty} (2\bar{x} - X)\phi(X)\,\mathrm{d}X$$

$$= \begin{bmatrix} 2\int_{-\infty}^{\underline{x}} \underline{x}\phi(X)\,\mathrm{d}X \\ -\int_{-\infty}^{\underline{x}} X\phi(X)\,\mathrm{d}X \end{bmatrix} + \begin{bmatrix} 2\int_{\underline{x}}^{\bar{x}} X\phi(X)\,\mathrm{d}X \\ -\int_{\underline{x}}^{\bar{x}} X\phi(X)\,\mathrm{d}X \end{bmatrix} + \begin{bmatrix} 2\int_{\bar{x}}^{\infty} \bar{x}\phi(X)\,\mathrm{d}X \\ -\int_{\bar{x}}^{\infty} X\phi(X)\,\mathrm{d}X \end{bmatrix}$$

$$= 2\eta_{DC} - \mu \quad \text{[use (2) viewing the above line by rows, rather than columns].} \tag{4}$$

Thus, the first moments of censored and folded distributions are related in a rather surprising manner, especially when we consider that the above proof is distribution-independent. Stated somewhat differently, given x_t, the expected future value of a stochastic process with a reflecting *mirror* barrier is related by a simple identity to the expected future value of that process with a reflecting *sticky* barrier. It is worth noting that (4) encompasses not only the doubly reflecting barrier, but also single reflecting barriers.[4]

2.2. *Explicit solutions when ε is Gaussian*

If $\varepsilon \sim \mathrm{N}(0, \sigma_\varepsilon^2)$ (Gaussian white noise) and if once again $X = x_t + k + \varepsilon_{t+1}$, then given any x_t, it follows that $X \sim \mathrm{N}(\mu, \sigma_\varepsilon^2)$ with pdf $\phi(X)$ and cdf $\Phi(X)$, where $\mu = x_t + k$. Then the explicit solution to (2) is given by (see Appendix A)

$$\eta_{DC} = \underline{x}\,\Phi(\underline{x}) + \mu[\Phi(\bar{x}) - \Phi(\underline{x})] - \sigma^2[\phi(\bar{x}) - \phi(\underline{x})] + \bar{x}[1 - \Phi(\bar{x})]. \tag{5a}$$

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[3] In the spirit of Leone, Nelson, and Nottingham (1961) who introduce 'the folded normal distribution' as a normal distribution with a single fold about zero (on the lower tail). This has been used in industrial practice in situations where measurements are recorded in absolute value (hence the fold at zero).

[4] With but a single reflecting barrier, secondary reflection is of course impossible. As such, the results below [(6b) and (6c)] *always* hold true.

Then, formally from (5a), or intuitively from (1), it follows that $\lim_{\mu \to \infty} \eta_{DC} = \bar{x}$ and $\lim_{\mu \to -\infty} \eta_{DC} = \underline{x}$ (as apparent in Fig. 3 below). Note that (5a) encompasses not only the doubly reflecting barrier, but also single reflecting barriers. For instance, to model a process that is bounded below by a sticky barrier (but with no upper bound), simply set $\bar{x} = \infty$ in (5a). Then x_{t+1} has a *lower censored* normal distribution, with mean

$$\eta_{LC} = \underline{x}\,\Phi(\underline{x}) + \mu[1 - \Phi(\underline{x})] + \sigma^2 \phi(\underline{x}). \tag{5b}$$

Similarly, to model a process bounded by a sticky upper barrier (but with no lower bound), simply set $\underline{x} = -\infty$ in (5a). Then x_{t+1} has an *upper censored* normal distribution, with mean

$$\eta_{UC} = \mu\,\Phi(\bar{x}) - \sigma^2 \phi(\bar{x}) + \bar{x}[1 - \Phi(\bar{x})]. \tag{5c}$$

Explicit solutions for reflecting *mirror* barriers are given by (4) so that by analogy:

$$\lambda_{DF} = 2\eta_{DC} - \mu \quad (DF \text{ denoting } \textit{doubly folded}), \tag{6a}$$

$$\lambda_{LF} = 2\eta_{LC} - \mu \quad (LF \text{ denoting } \textit{lower folded}), \tag{6b}$$

$$\lambda_{UF} = 2\eta_{UC} - \mu \quad (UF \text{ denoting } \textit{upper folded}). \tag{6c}$$

The reflecting *mirror* barrier has a continuous-time analogue. To illustrate the point, let $\{X(t), t \geq 0\}$ denote the absolute Brownian motion $\mathrm{d}X = a\mathrm{d}t + b\mathrm{d}z$ with drift a and instantaneous standard deviation b, where z is a Wiener process. Then, for all $t > s$,

$$X(t) \sim \mathrm{N}(\mu, \sigma^2) \quad \text{with} \quad \text{pdf}\,\phi(\cdot), \text{cdf}\,\Phi(\cdot), \mu = X(s) + a(t - s),$$

$$\sigma^2 = b^2(t - s).$$

Let $\{Y(t), t \geq 0\}$ denote $\{X(t), t \geq 0\}$ reflected at the origin. Then $\mathrm{E}[Y(t)] = \lambda_{LF}$. Thus, in the simple text example of a Wiener process ($a = 0$, $b = 1$, $X(0) = 0$), one obtains $\mathrm{E}[Y(t) \mid s = 0] = 2t\phi(0) = \sqrt{2t/\pi}$, as per say Kannan (p. 233). Of course, the solutions provided here are more general.

3. An application: Exchange rate target zones

An exchange rate target zone is simply an announced band within which the exchange rate is allowed to float. Target zone intervention is defined here to be purely marginal intervention: that is to say, intervention that is triggered only when the exchange rate reaches the edge of the band. Such zones are interesting because their very presence influences the exchange rate through expectations, even when the band is not binding – that is, even when the exchange rate lies inside its band. We shall consider[5] a perfectly credible target zone under two different regimes:

Free float with target zone: The exchange rate is allowed to float freely inside the band.

Managed float with target zone: There is intra-marginal intervention as well. The total sum of intervention will now be intervention inside the band (the managed float) plus a further intervention policy at the edge of the band (the target zone).

For each of the above regimes, we consider two types of target zone intervention:

Reflecting sticky barriers: Should the exchange rate reach one of its bounds, the authority intervenes by an amount just sufficient to keep the exchange rate at that bound. This is consistent with the notion of infinitesimal intervention at the margin.

Reflecting mirror barriers: Rather than just keeping the exchange rate at its bounds, intervention now forces the exchange rate back *into* the band. Moreover, the larger the shock, the more the exchange rate will be forced back into the band, other things being equal.

In the international finance literature, it is now widely accepted that it is difficult to outperform a random walk in forecasting nominal exchange rates

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[5] An explicit model of intervention, such as that presented here, will require that the underlying shock-generating process $\{\varepsilon_t\}$ does not change following the introduction of the zone, that the intervention policy is known, and that the authority can set any desired exchange rate by means of direct intervention in the foreign exchange market.

under a free float.[6] In essence, a random walk is still the benchmark to beat. As such, let the free float process be given by[7]

$$x_{t+1}^* = x_t^* + k + \varepsilon_{t+1}, \quad \varepsilon \sim \mathrm{N}(0, \sigma_\varepsilon^2)$$

$$\begin{cases} x_{t+1}^* \sim \mathrm{N}(\mu^{FF}, \sigma_\varepsilon^2) \quad \text{with} \quad \mathrm{pdf}\,\phi^{FF}(\cdot),\ \mathrm{cdf}\,\Phi^{FF}(\cdot), \\ \mu^{FF} = x_t^* + k. \end{cases} \tag{7}$$

With leaning-against-the-wind, a managed float would then yield:

$$x_{t+1}^* = x_t^* + k - \rho(x_t^* - \hat{x}) + \varepsilon_{t+1}$$

$$\begin{cases} x_{t+1}^* \sim \mathrm{N}(\mu^{MF}, \sigma_\varepsilon^2) \quad \text{with} \quad \mathrm{pdf}\,\phi^{MF}(\cdot),\ \mathrm{cdf}\,\Phi^{MF}(\cdot), \\ \mu^{MF} = (1-\rho)x_t^* + \rho\hat{x} + k, \end{cases} \tag{8}$$

where $\rho \in [0, 1]$ measures the degree of leaning, and $\hat{x}$ denotes the authority's desired exchange rate.

We wish to derive the expected future exchange rate under each regime, conditional on Ω_t (the information set at time t).

The free float (FF) solution is given instantly by (7) as

$$\mathrm{E}[x_{t+1}^* \mid \Omega_t^{FF}] = \mu^{FF}. \tag{9}$$

The managed float (MF) solution is given instantly by (8) as

$$\mathrm{E}[x_{t+1}^* \mid \Omega_t^{MF}] = \mu^{MF}. \tag{10}$$

[6]The seminal Meese and Rogoff paper (1983) springs to mind, amongst many others.

[7]This is consistent with Krugman models where the free float exchange rate also follows a random walk. Nevertheless, we stress an important distinction here. Barnett (1992) and King et al. (1992) have recently shown that exchange rates are able to display randomness unrelated to fundamentals. More strikingly, Flood and Rose (1993, p. 3) are 'driven to the conclusion that the most critical determinants of exchange rate volatility are not macroeconomic'. More generally, tests for excess volatility in financial markets suggest that asset prices are affected by more than fundamentals – an empirical result consistent with the theoretical literature on fads, bandwagon effects, and rational stochastic speculative bubbles. Since Krugman models only capture fundamental sources of randomness, it seems unlikely that they capture the full stochastic nature of exchange rates.

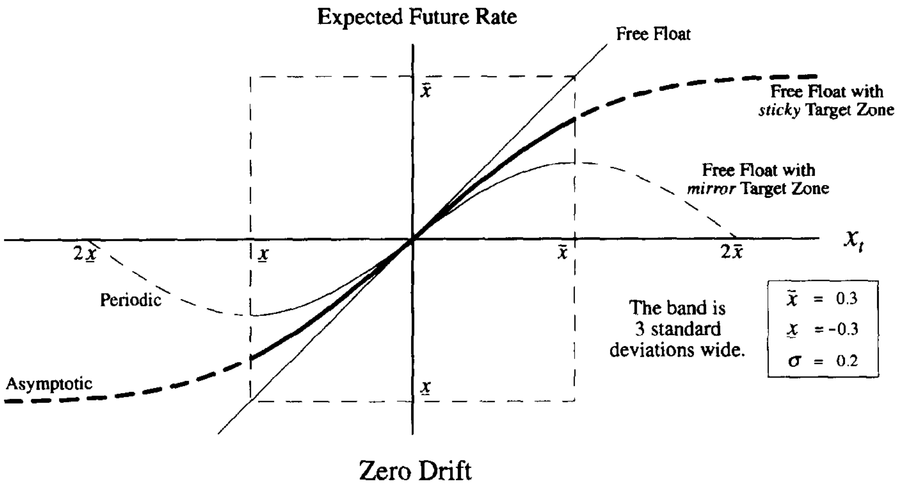

Fig. 3. Free float and target zone.

The free float *with* target zone solution is given instantly by (5a) and (6a) as

$$\mathrm{E}[x_{t+1} \mid \Omega_t^{FFTZ}] = \begin{Bmatrix} \eta_{DC} & \text{if} & \underline{x}, \bar{x} \text{ are } \textit{sticky} \\ \lambda_{DF} & \text{if} & \underline{x}, \bar{x} \text{ are } \textit{`mirrors'} \end{Bmatrix} \tag{11}$$

with $\{\mu = \mu^{FF}, \quad \phi = \phi^{FF}, \quad \Phi = \Phi^{FF}.$

The managed float *with* target zone solution is given instantly by (5a) and (6a) as

$$\mathrm{E}[x_{t+1} \mid \Omega_t^{MFTZ}] = \begin{Bmatrix} \eta_{DC} & \text{if} & \underline{x}, \bar{x} \text{ are } \textit{sticky} \\ \lambda_{DF} & \text{if} & \underline{x}, \bar{x} \text{ are } \textit{`mirrors'} \end{Bmatrix} \tag{12}$$

with $\{\mu = \mu^{MF}, \quad \phi = \phi^{MF}, \quad \Phi = \Phi^{MF}.$

The target zone solutions (11) and (12) each define a nonlinear first-order expectational difference equation $\mathrm{E}[x_{t+1} \mid \Omega_t^{-TZ}] = f(x_t \mid \cdot)$, where f is a function defined by (11) or (12), as appropriate. Plotting $\mathrm{E}[x_{t+1} \mid \Omega_t^{-TZ}]$ against x_t then yields a nonlinear S-shaped curve[8] (when viewed within the band). Fig. 3 illustrates for a free float regime, while Fig. 4 illustrates for a managed float.

[8] This relationship between the expected future rate and the present rate can also be plotted with Krugman-style models and doing so yields similar results – see for instance Rose and Svensson (1991, p. 6, Fig. 2).

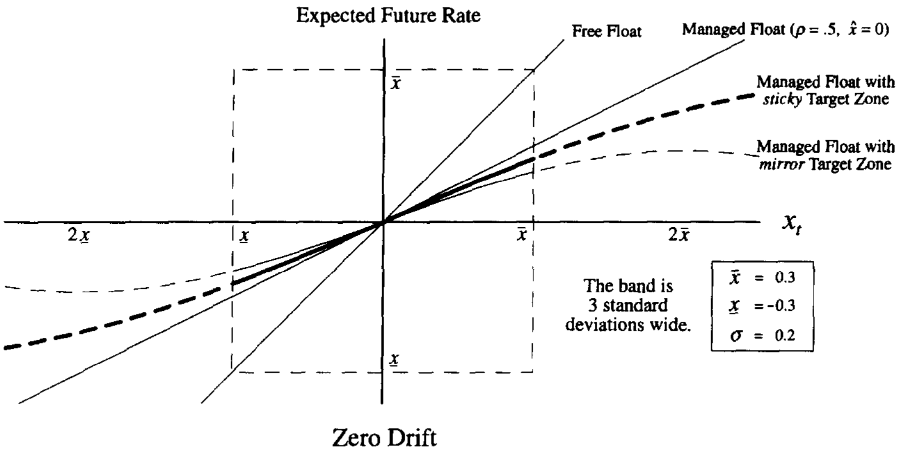

Fig. 4. Managed float and target zone.

Because the S-curves in Fig. 3 have slope less than one and lie between the free float line and the horizontal axis, the standard stabilising properties of a perfectly credible target zone follow. Similarly, in Fig. 4, the S-curves lie between the managed float line and the horizontal axis. Moreover, because the S-curves are now viewed in intertemporal space, convergence properties can be illustrated as well. This is discussed further in Rose (1995) where the empirical implications of convergence are discussed. By contrasting Fig. 3 with Fig. 4, it is interesting to observe that with leaning-against-the-wind, the nonlinearity of the two target zone solutions (within the band) is considerably reduced.[9] Indeed, the 'S'-curves in Fig. 4 are essentially linear within the band. Imperfectly credible bands are beyond the scope of this paper. Nevertheless, we briefly note that such regimes are often modelled as some convex combination of the perfectly credible solution and the crash solution (as per Krugman, 1991). If so, an imperfectly credible regime follows as a natural extension from the perfectly credible solution derived above.

Recently, the target zone literature has attracted criticism (a) because it assumes that markets are 'excessively' rational by imposing saddle path solutions (see Krugman and Miller, 1993) and (b) because such models are fundamental asset pricing models (see footnote 7; also see Williamson, 1993, who describes the breakdown in the belief that exchange rates are driven primarily by fundamentals). By contrast, the statistical approach illustrated in this paper circumvents such criticisms, and it does so without any need for differential equations or smooth pasting conditions. Despite these differences, it would appear that the central stability theorems of the literature are robust.

[9]This result is consistent with Svensson (1992).

4. Coda

This note has served two roles. First, it has illustrated a surprising statistical identity that exists between folded and censored distributions, and we show that this is relevant to random processes bounded by reflecting barriers. Second, by using the derived framework, the paper provides a novel and conceptually different approach to modelling exchange rate target zones that circumvents many of the theoretical problems associated with the existing target zone literature.

Appendix A

Let $Z \sim \mathrm{N}(0,1)$ with pdf $h(z)$ and distribution function $H(z)$, and let $X \sim \mathrm{N}(\mu, \sigma^2)$ with pdf $\phi(x)$ and distribution function $\Phi(x)$.

If $Z = (X - \mu)/\sigma$ one can show that

(a) $\phi(x) = \dfrac{h(z)}{\sigma}$,

(b) $\Phi(x) = H(z)$,

(c) by direct integration $\displaystyle\int_{\underline{z}}^{\bar{z}} z\,h(z)\,\mathrm{d}z = h(\underline{z}) - h(\bar{z})$.

By using the change of variable $x = \mu + \sigma z$, it follows that

$$\int_{\underline{x}}^{\bar{x}} x\,\phi(x)\,\mathrm{d}x = \int_{\underline{z}}^{\bar{z}} (\mu + \sigma z)\frac{h(z)}{\sigma}(\sigma\,\mathrm{d}z) \qquad \text{applying (a),} \quad \text{with } \underline{z} = \frac{\underline{x} - \mu}{\sigma}\cdots,$$

$$= \mu\int_{\underline{z}}^{\bar{z}} h(z)\,\mathrm{d}z + \sigma\int_{\underline{z}}^{\bar{z}} z h(z)\,\mathrm{d}z$$

$$= \mu[H(\bar{z}) - H(\underline{z})] - \sigma[h(\bar{z}) - h(\underline{z})]$$

applying (c),

$$= \mu[\Phi(\bar{x}) - \Phi(\underline{x})] - \sigma^2[\phi(\bar{x}) - \phi(\underline{x})]$$

applying (b) and (a).

Appendix B

A note on secondary and higher-order reflection

Let $s = \bar{x} - \underline{x}$ denote the size of the band, and let $x_0 = (\underline{x} + \bar{x})/2$ denote the centre of the band. Then, *any* outcome X can always be decomposed as follows:

$$X = x_0 + s \cdot n + c \quad \text{where} \quad n = 0, \pm 1, \pm 2, \dots, \quad \text{and} \quad |c| \leq s/2.$$

In the presence of lower ($\underline{x}$) *and* upper ($\bar{x}$) reflecting *mirror* barriers, secondary reflection may occur. If $n = 0$, there will be no reflection at all. In Section 2, it was assumed that $|n| < 2$ (i.e., no secondary reflection). This appendix expands the analysis to nth-order reflection (the 'particle' is reflected n times) when $\phi(X)$ is normal. The reflected process $\{x_t\}$ will now be defined recursively by $x_{t+1} = g(X)$, where $g\colon \mathbb{R} \to [\underline{x}, \bar{x}]$ is defined by

$$g(X) = x_0 + (-1)^n c \quad \text{where} \quad n = \text{Round}\left[\frac{X - x_0}{s}\right]$$

$$\text{and} \quad c = X - x_0 - s \cdot n$$

(the function Round$[x]$ gives the integer closest to x).

This replaces Eq. (3). It follows that g is a periodic function with period $2s$.

Section 2 showed that $\lambda_{DF} = 2\eta_{DC} - \mu$ assuming $|n| < 2$. As above, if $\phi(X)$ is normal, this assumption will be reasonable if (i) $\mu \in [\underline{x}, \bar{x}]$ and (ii) $s \geqslant 3\sigma$. We now illustrate how λ_{DF} can be used when $\mu \notin [\underline{x}, \bar{x}]$. To do so, note that *any* μ can be decomposed as follows:

$$\mu = x_0 + s \cdot n + c.$$

Let $\Theta[\mu | \sigma, \underline{x}, \bar{x}]$ denote the pdf of the generalised doubly folded normal density function derived from a normal density function with mean μ and variance σ^2 (see Fig. 2). Then, using simple graphical analysis (use Fig. 2), one can show that

$$\Theta[\mu \mid \cdot] = \Theta[g(\mu) \mid \cdot] \quad \text{where} \quad g(\mu) \in [\underline{x}, \bar{x}], \text{ thus satisfying (i).}$$

In summary then, when $\phi(X)$ is normal, Eq. (6a) may be stated:

$$\lambda_{DF} = 2\eta_{DC} - \mu \quad \text{if} \quad \text{(i) } \mu \in [\underline{x}, \bar{x}] \quad \text{and} \quad \text{(ii) } s \geq 3\sigma. \tag{13}$$

If $\mu \notin [\underline{x}, \bar{x}]$, simply replace μ with $g(\mu)$ throughout the RHS of Eq. (13).

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